Very simple models, the self-modifying automata and chain of self-modifying automata, can explain self referential properties of living beings.

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ABSTRACT

Very often, the living beings seem able to change their functioning when the external conditions vary.

In order to study this property, we devised abstract machines whose internal organisation change whenever the external conditions vary.

The internal organisations of these machines (or program), as simple as possible, are functions of discrete variable. We called such machine **self modifying automata**.

These machines after any transient steps stabilise when they go indefinitely trough a loop called p-cycle or limit cycle of length p.

More often than not, the p-cycle is equal to one and is called fixed point.

In this case the external value (v) can be considered as the index of function f such as : $f_v(v) = v$ and the machine has the property of **self replication** and to be **self referential**.

Many authors, in computer and natural science, consider that self referential objects are a main concept in comprehension of perception, behaviour and associations.

In the third part, we studied chains of automata. Only one automaton changes its internal organisation at each step. Chains of automata have better performances than single self-modifying automata : Higher frequency of fixed point and a shorter transient length. The performances of the chains of automata improve when the value of their internal states increases whereas the performances of single automata decrease.

1 - Introduction.

1.1 - Property of livings beings to change their internal organisation when the external conditions vary.

Living organisms, both the functional structures which are the parts of living beings, and the society of living animals and plants, are able to change their internal organisation whenever the external conditions vary.

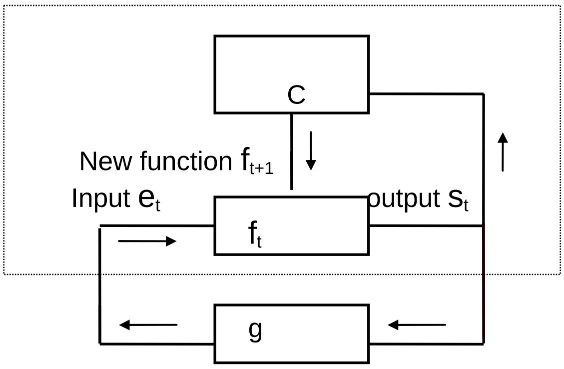
- If a Dove's prism is placed in front of both eyes of a man, the visual field is inverted. After a few weeks, the field of vision is perceived normally again (Gonshor et al.,1976).
- Ants, which very quickly find their bearings, are able to find the shortest route in an extremely complicated man-made maze (Chauvin, 1975)..
- Paramecium is able to learn (Gelber, 1958).
- A plant modifies its growth when a part has been rubbed and cut Desbiez et al., 1991).
- The immune system responds specifically to a new synthetic molecule (Lodish et al.,1995)

1.2 - Modelling this property.

In order to study this property, we devised abstract machines, as simple as possible whose internal organisation change whenever the external conditions vary. (Fig. I).

The internal organisation of these machine is a function (f) of discrete variable, the input is the value of the variable and the output is the value of this function.

Figure 1



External process

A function C, build at random at the beginning and once for all, assigns to each value of the input (or the output) a function f. This function C is the inverse of the function of *Gödelisatio*n which assigns to each program a number (Nagel et al., 1989). The machine is connected to an external function g. We called this machine a **self-modifying automaton** and this machine is a deterministic finite automaton (D.F.A.) without final states. The initial state is randomly chosen. This D.F.A with output is a modification of the Mealy machine (Hpcroft & Ulmann, 1972).

After any transient step, the machine stabilises when it goes indefinitely trough a loop called p-cycle or limit cycle of length p.

More often than not, the p-cycle is equal to one and is called fixed point. In this case the external value (v) is the index of function f such as : $f_v(v) = v$ and the machine has the property of **self replication** (Von Neumann, 1966; Myhill, 1970; Codd, 1968; Greussay, 1988) and to be **self referential** (Thatcher, 1965).

Many authors, in computer (Ashby, 1961) and natural science (Maturana & Varela, 1981; Zeleny, 1981) consider that self reference is a fundamental concept in comprehension of the mechanisms of perception, behaviour and associations in the brain (Bartlett &Suber, 1987).

1.3 - Comparison with others machines and automata.

- Some authors have studied dynamics of automata where connections and truth tables are randomly chosen at the beginning and once for all, which implies that internal rules remain fixed (quenched model) (Kaufmann, 1969 ; Fogelman Soulié, 1985).
- Other authors have studied dynamics of automata of which rules are randomly modified at each step. (Annealed model) (Derrida & Stauffer, 1986)
- Lastly, many authors, in order to simulate memory and pattern recognition in the brain, devised machine whose internal functioning is modified in order to minimize the difference between the output and goal value. Let's quote: (Adaline Mattson, 1959; Widrow & Hoff, 1960), the association network (Anderson, 1972; Kohonen, 1972) the Cognitron (Fukushima, 1975) the Hopfield's association Network (Hopfield, 1982) and the multi layered Back-propagation association network (Rumelhart et al., 1986)

2. Self-modifying automata

2.1. Definitions

We define here «Self-modifying automata» and show the properties of their dynamics.

Notations

Let be given :

- $A \subset \mathrm{N}$, a finite subset of N
- $F_A = \{f : A \to A\}$, a set of functions which map A into itself.

$F_A(A) \subseteq A$ is the set of *images* of A through functions of F_A :

$$F_A(A) = \bigcup_{f \in F_A} f(A)$$

- C : $A \to F_A$ a function .

2.1.1. Definition 1

An *automaton* **a** is defined as an n-uple (S, I, O, F, G) where:

- S is the set of *internal states* of **a**,
- I is the set of *inputs* of **a**,
- O is the set of *outputs* of **a**,
- F : $I \times S \to S$ is the *transition function* of **a**,
- G: $I \times S \to O$ is the *output function* of **a**

Definition 2

We will say that $\mathbf{a}_{\mathbf{sm}}$ = (A, F_A, C, g) is a *self-modifying automaton* (Fig.2) iff:

- $g \in F_A$,
- S = F_A,
- I = O = A,
- output function G: $A \times F_A \to A$ is defined by:

$$\forall (e_t, f_t) \in A \times F_A, \quad s_t = G(e_t, f_t) = f_t(e_t) \tag{1}$$

- transition function F : $A \times F_A \to F_A$ is defined by:

$$\forall (e_t, f_t) \in A \times F_A, \quad f_{t+1} = F(e_t, f_t) = C[s_t] \tag{2}$$

- $e_{t+1} = g(s_t)$ (3)

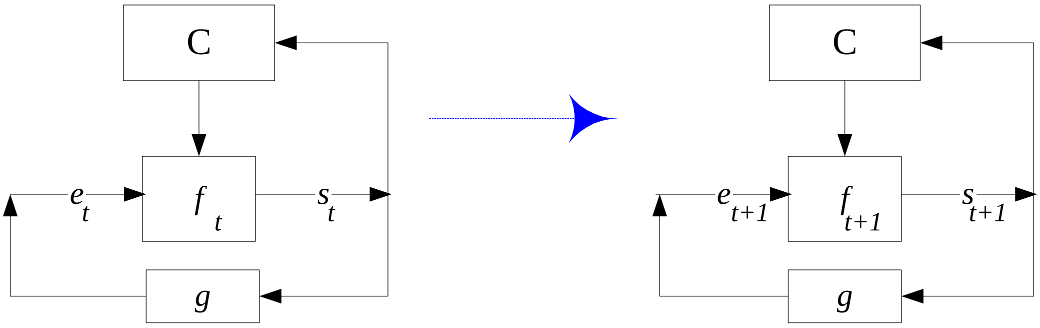


Figure 2

2.1.2. Definition 3

We will say that *self-modifying automaton* **a**$_{\mathbf{sm}}$ = (A, F_A, C, g) is self-*connected* (Fig.3) iff $g = Id$, i.e. :

- $e_{t+1} = s_t$ (4)

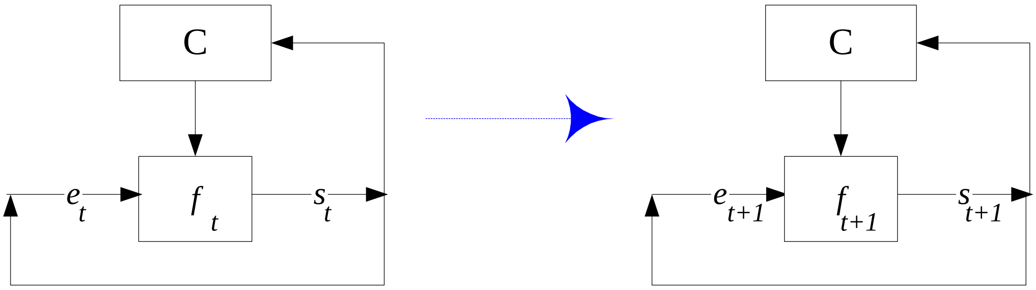


Figure 3

2.1.3. Definition 4

The *trajectory* of self-modifying automaton **a**$_{\mathbf{sm}}$ is defined as the set of successive (state, output) pairs $\{(f_1, s_0), \ldots, (f_t, s_{t-1}), (f_{t+1}, s_t), \ldots\}$,calculated through equations (1), (2), (3) or (4) :

$$\left| \begin{array}{l} f_{t+1} = C[s_t] \\ e_{t+1} = s_t \text{ or } e_{t+1} = g(s_t) \\ s_t = f_t(e_t) \end{array} \right.$$

(f_1, s_0) is called the *initial point* of the trajectory and is usually set at random. F_A and C are fixed for any given automaton **a**$_{\mathbf{sm}}$ $=(A, F_A, C, g)$: *drawing* **a**$_{\mathbf{sm}}$ *at random* will mean drawing at random A, F_A and C.

We will denote (f_{t+1}, s_t) = **a**$_{\mathbf{sm}}$ (f_t, s_{t-1}) (5)

2.1.4. Theorem 1

Any trajectory of a self-modifying automaton **a**$_{\mathbf{sm}}$ converges to a limit cycle.

Proof: since A is finite, F_A is finite too, and thus the cardinal number of set $F_A \times A$ is finite: the number of points (f_t, s_{t-1}) is thus finite, so that the trajectory must, after a finite time, pass through a point (f_t, s_{t-1}) it has already been into before.

If the length of the limit cycle is *p*, we will call the limit cycle a p-cycle. We will call transient time, the number τ of points (f_t, s_{t-1}) before automaton **a**$_{\mathbf{sm}}$ enters its limit cycle (Fig.4).

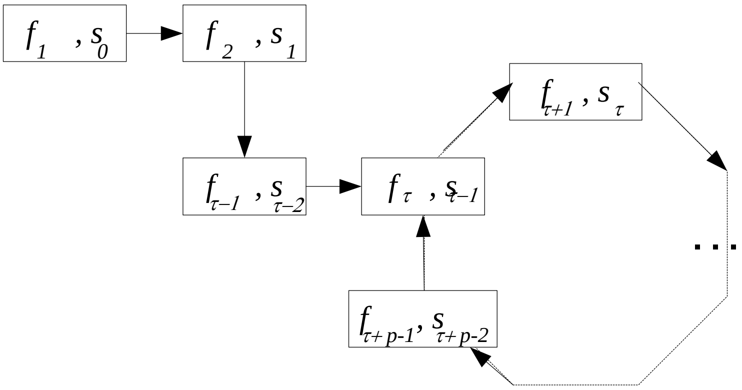


Figure 4

2.1.5. Definition 5

We will say that self-modifying automata **a** = (A, F_A, C, g) is *equivalent* to **a'**=$(A, F_{A'}, C', Id)$ iff $\{\ldots,(f_{t+1}, s_t),\ldots\}$ and $\{\ldots,(f'_{t+1}, s'_t),\ldots\}$ are the trajectories of **a** and **a'**then:

$$\forall t, \quad f_t = f'_t \circ g, \quad s_t = s'_t \tag{6}$$

2.1.6. Theorem 2

Any self-modifying automaton **a** = (A, F_A, C, g) is equivalent to a self-connected self-modifying automaton **a'** = $(A, F_{A'}, C', Id)$:

Proof :

By recurrence, it is sufficient to show that if (f_t, s_{t-1}) and (f'_t, s'_{t-1}) satisfy condition (6), then so do (f_{t+1}, s_t) and (f'_{t+1}, s'_t).

From equations (1) to (3):

$s'_t = f'_t \circ g(s'_{t-1}) = f'_t \circ g(s_{t-1}) = f_t(s_{t-1})$ because of the recurrence assumptions.

Hence: $s'_t = s_t$

$$f'_{t+1} \circ g = C' \circ f'_t \circ g(s'_{t-1}) \circ g = C' \circ f'_t \circ g(s_{t-1}) \circ g = C' \circ f_t(s_{t-1}) \circ g$$

$C' \circ f_t(s_{t-1}) \circ g = C'(s_t) \circ g = C(s_t) \circ g = f_{t+1} \circ g$ if $C = C'$

Thus **a** = (A, F_A, C, g) is equivalent to **a'** = (A, F_A, C, Id).

In the following, we will thus only use self-connected self-modifying automata, and will denote, for the sake of simplicity, **a** = (A, F_A, C)

2.2. Properties

Let be given **a** = (A, F_A, C) a self-connected self-modifying automaton and let us denote :

$$m = card[A \times C(A)] \tag{7}$$

where $C(A)$ is the image of A in F_A : if A has *n* elements, than $m \leq n^2$.

From Theorem 1, we know that, from any initial condition (f_1, s_0), automaton **a** enters after a finite time τ into a limit cycle of length *p*. Let *k* be the first time when automaton **a** passes through a state (f_t, s_{t-1}) it has been into before: we will say that **a** *stabilises* at time *k*.

$$k = \tau + p$$

2.2.1. Theorem 3

The probability that automaton **a** = (A, F_A, C) stabilises at time *k* is given by:

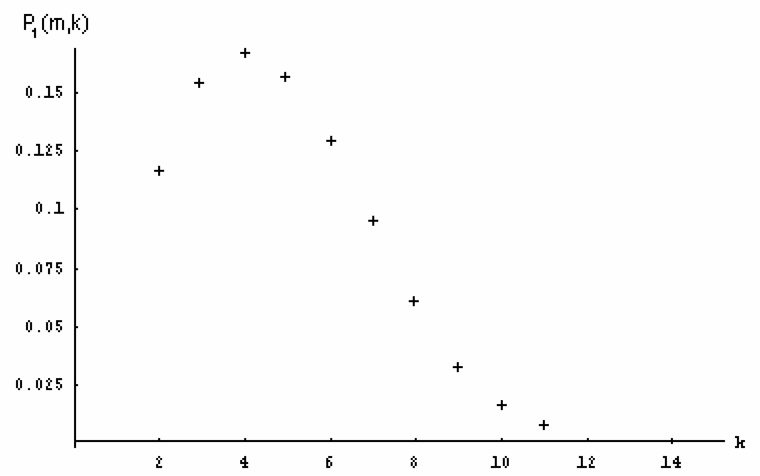
$$P_1(m,k) = \frac{A_m^k . k}{m^{k+1}} \tag{8}$$

where A_m^k is the number of arrangements of m objects *k* by *k*.

This result has been demonstrated in (Moulin, 1992).

We show (Fig. 5) the value of $P_1(m,k)$ given by (8), for various values of k : note that $P_1(m,k)$ is maximum for $k = \sqrt{m}$.

Figure 5 : $P_1(m,k)$ as a function of k with m = 16



2.2.2. Theorem 4

The probability that automaton **a** = (A, F_A, C) enters into a limit cycle of length *p* is given by:

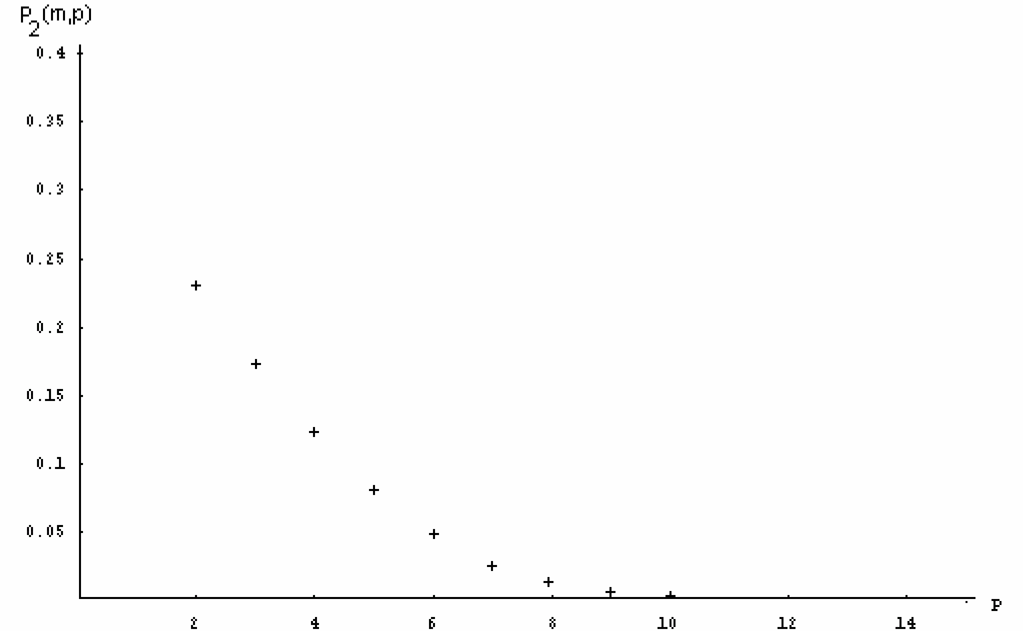
$$P_2(m,p) = \sum_{k=p}^{m} \frac{A_m^k}{m^{k+1}} \tag{9}$$

This result has been demonstrated in (Moulin, 1992). It is sufficient to see that:

$$P_2(m,p) = \sum_{k=p}^{m} \frac{P_1(m,p)}{k}$$

We show (Fig. 6) $P_2(m,p)$, as computed from (9) for various values of p .

Figure 6 : $P_2(m,p)$ as a function of p with m = 16.



2.3. Performances

We have seen in theorem 1 that any self-modifying automaton enters, after a finite transient time τ into a limit cycle of finite length *p*.
We are interested in automata which stabilise fast into short cycles, i.e. in automata with small *k*: e.g. $\tau = 0, p = 1$. In such a case, the limit cycle defines a function, which is the function which the automaton has «self-programmed».
As can be seen (Fig. 5) and (Fig. 6), *k* and *p* grow when *m* grows : this is a non desirable property, if one thinks of biological systems, where the state space probably is of large dimension *m*. We will define in section 3 another class of automata, which stabilise much faster for large *m*.
We will now define performances criteria for populations of automata : these will allow us to compare the performances of various classes of automata.
Let thus be $\wp = \{(A, F_A, C_i) / i \in \Im\}$ and let us denote:

- the expected limit cycle length, transient length and stabilisation time of automata **a** in $\wp$:

$$E_\wp(p) = P(p = 1) + 2P(p = 2) + 3P(p = 3) + ... \qquad (10)$$
$$E_\wp(\tau) = P(\tau = 1) + 2P(\tau = 2) + 3P(\tau = 3) + ...$$
$$E_\wp(k) = P(k = 1) + 2P(k = 2) + 3P(k = 3) + ...$$

- $R_\wp$ the probability that an automaton in population $\wp$ has a limit cycle of length $p = 1$

$$R_\wp = \frac{\sum_{a \in \wp} \sum_{(f_1, s_0) \in F_A \times A} 1_{(p=1\}}}{\sum_{a \in \wp} \sum_{(f_1, s_0) \in F_A \times A} 1_{(p \geq 1\}}} \qquad (11)$$

Definition 6

We will say that population $\wp_1$ is better than population $\wp_2$ according to criterion *C* iff :

$$C_{\wp_1} \leq C_{\wp_2}$$

For example, for criterion $E(p)$ $\quad E_{\wp_1}(p) \leq E_{\wp_2}(p)$

Let us define a nested family of populations $\wp_m$ as follows:

1. $n = 2$
2. $A = [0, n-1]$ and $F_A = \{f : A \to A\}$
3. for all *C* : $A \to F_A$, do
4. compute *m*
5. for all *m* possible initial conditions (f_1, s_0) in $F_A \times A$, compute the trajectory of automaton (A, F_A, C), measure p, τ, k and count occurrences of event $\{p = 1\}$.
6. $n = n + 1$, as long as $n \leq 4$ goto 2

We show (Fig. 7-a) criterion $R_{\wp_m}$ and $E_{\wp_m}(p)$ as a function of *m*.

Figure 7-a: $R_{\wp_m}$ as a function of *m*

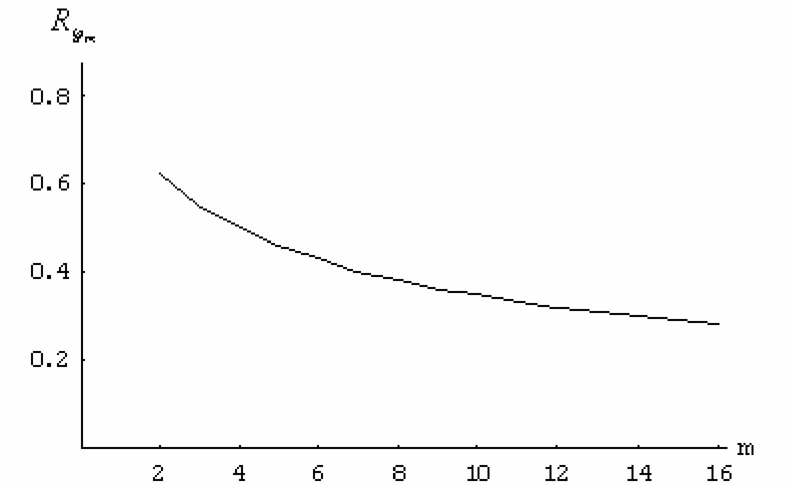
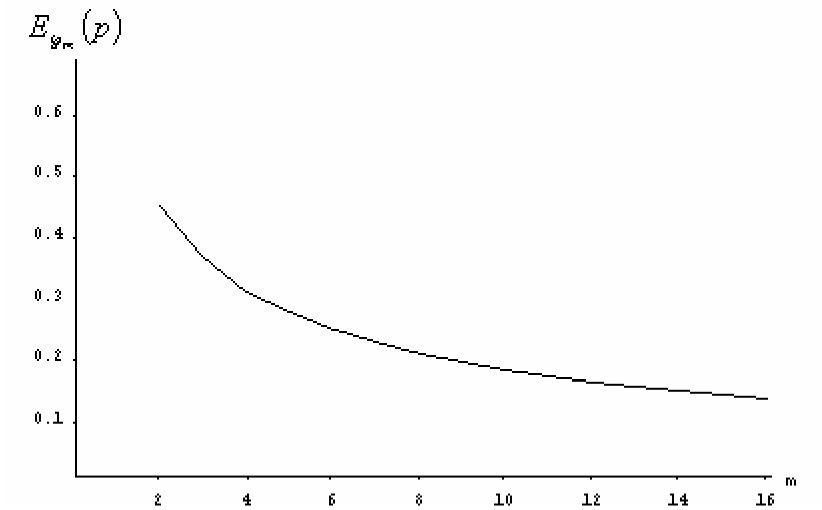


Figure 7-b: $E_{\wp_m}(p)$ as a function of *m*



We show (Fig. 7-b) that self-modifying automata have a decreasing probability of stabilising in a fixed point when *m* increases.

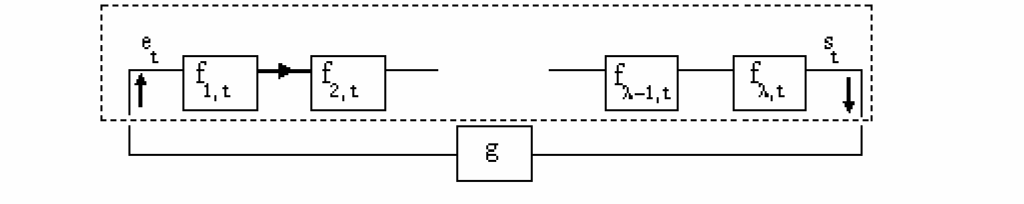
3 - Chains of self-modifying automata.

Notation.

A series of λ self-modifying automata is put in place (Fig. 8).

Each self-modifying automaton is characterized by an index i, with $i \in [0, \lambda - 1]$.

Figure 8



Let be given :

- P_A = { p : $F_A^\lambda \rightarrow F_A$} a set of functions which maps F_A^λ onto F_A.
- p$(f_{\lambda-1}, \ldots, f_0) = (f_{\lambda-1} \circ \ldots \circ f_0)$ (12)

- P$_A$ (F_A^{λ}) $\subseteq F_A$ is the set of *images* of F_A^{λ} through functions of P$_A$:

$$\mathrm{P}_A\,(F_A^{\lambda-1},...,F_A^{0}) = \bigcup_{p \in P_A} p(f_{\lambda-1},\dots,f_0)$$

3.1 - Definition 1

A *chain of automata* **Ca** is defined as an n-uple (S, I, O, F, G) where:

- *S* is the set of *internal states* of **Ca**,
- *I* is the set of *inputs* of **Ca**,
- *O* is the set of *outputs* of **Ca**,
- *F* : $I \times S \to S$ is the *transition function* of **Ca**,
- *G*: $I \times S \to O$ is the *output function* of **Ca.**

3.2 - Definition 2

We will say that **Ca$_{\mathbf{sm}}$** = (A, F_A, C, g) is a *chain of self-modifying automaton* (Fig.8) iff:

- $g \in F_A$,
- S = F_A^{λ} ,
- I = O = A,
- output function $G\colon A \times P_A(\mathrm{F}_A^{\lambda}) \to A$ defined by :

$$\forall(e_t, p_t(f_{\lambda-1},\dots,f_{\lambda-1})) \in A \times P_A(F_A^{\lambda}), \qquad s_t = G(e_t, p_t(f_{\lambda-1.t},\dots,f_{0,t})) = [p_t(f_{\lambda-1.t},\dots,f_{0,t})](e_t) \tag{13}$$

- transition function $F\colon A \times F_A^{\lambda} \to F_A^{\lambda}$ is defined by:

$$\forall(e_t,(f_{\lambda-1,t},\dots,f_{0,t})) \in A \times F_A^{\lambda},$$
$$(f_{\lambda-1,t+1},\dots,f_{0,t+1}) = F(e_t,(f_{\lambda-1,t},\dots,f_{0,t}))$$
$$(f_{\lambda-1,t+1},\dots,f_{i,t+1},\dots,f_{0,t+1}) - (f_{\lambda-1,t},\dots,f_{i,t},\dots,f_{0,t}) = (0,\dots,\ f_{i,t+1} - f_{i,t}\ ,\dots,0) \tag{14}$$

with $f_{i,t+1}$ = $C_i(s_t)$ and $i = e_t$.

- $e_{t+1} = g(s_t)$ (15)

3.3 - Definition 3

The *trajectory* of a chain of self-modifying automaton **Ca$_{\mathbf{sm}}$** is defined as the set of successive (state, output) pairs $\{((f_{\lambda-1,1},\dots,f_{0,1}),s_0),\dots,((f_{\lambda-1,t},\dots,f_{0,t}),s_{t-1}),((f_{\lambda-1,t+1},\dots,f_{0,t+1}),s_t),\dots.\}$,calculated through equations (12), (13), (14) and (15) :

$$\left|\begin{array}{l} (f_{\lambda-1,t+1},\dots,f_{0,t+1}) = F(e_t,(f_{\lambda-1,t},\dots,f_{0,t})) \\ e_{t+1} = g(s_t) \\ s_t \ = f_{\lambda-1,t}\circ\dots\circ f_{0.t}(e_t) \end{array}\right.$$

$((f_{\lambda-1,1},\dots,f_{0,1}),s_0)$ is called the *initial point* of the trajectory and is usually set at random. P_A, F_A and C are fixed for any given automaton **Ca$_{\mathbf{sm}}$** = (A, F_A, C, g) : *drawing* ***Ca$_{\mathbf{sm}}$*** *at random* will mean drawing at random A , P_A, F_A and C .

We will denote $((f_{\lambda-1,t+1},\dots,f_{0,t+1}),s_t)$ = **Ca$_{\mathbf{sm}}$**$((f_{\lambda-1,t},\dots,f_{0,t}),s_{t-1})$ (16)

Theorem 1

Any chain of a self-modifying automaton **Ca$_{\mathbf{sm}}$** has an equivalent self-modifying automaton.
Proof: Let's consider the equality (12) : p $(f_{\lambda-1},\dots,f_0) = (f_{\lambda-1}\circ\dots\circ f_0)$.

Let be given : f_t^{*} = $(f_{\lambda-1,t}\circ\dots\circ f_{0,t})$. (17)

The equality (16) becomes $((f_{t+1}^{*}, s_t)$ = **Ca$_{\mathbf{sm}}$**(f_t^{*}, s_{t-1}) which can be identified with the equality (5) : (f_{t+1}, s_t) = **a$_{\mathbf{sm}}$** (f_t, s_{t-1}) which characterises a self-modifying automaton

3.4 - Dynamics.

Let be given :

- α = card $[F_A]$
- α^λ = card $[F_A^\lambda]$
- m = card $[F(A \times F_A^\lambda)]$

therefore m $\leq \alpha^\lambda$

The equality (14) : $(f_{\lambda-1,t+1},\ldots,f_{i,t+1},\ldots,f_{0,t+1})$ - $(f_{\lambda-1,t},\ldots,f_{i,t},\ldots,f_{0,t})$ = (0,..., $f_{i,t+1} - f_{i,t}$,..., 0) shows that only one automaton of the chain has its function modified at each step t.
The vector (0,..., $f_{i,t+1} - f_{i,t}$,..., 0) is equal to (0, ...,0) if $f_{i,t+1} = f_{i,t}$.

If α =2 , $(f_{\lambda-1,t+1},\ldots,f_{i,t+1},\ldots,f_{0,t+1})$ - $(f_{\lambda-1,t},\ldots,f_{i,t},\ldots,f_{0,t})$ is the Hamming's distance between two consecutive internal states.

One doesn't known any algebraic expression giving the frequency of τ , number of transient points and the frequency of p, number of points of the p-cycle from the values α and λ (Kréveras, 1993). Therefore, we obtain results about chain of self-modifying automata only by simulation.

3.5 - Results of simulations.

We give the value of $E_\wp(p)$ (Fig. 9-a) and $E_\wp(\tau)$ (Fig. 9-b) when α or λ varies. In this example, $\alpha = \lambda$.

Figure 9-a : $E_\wp(p)$ as a function of α or λ

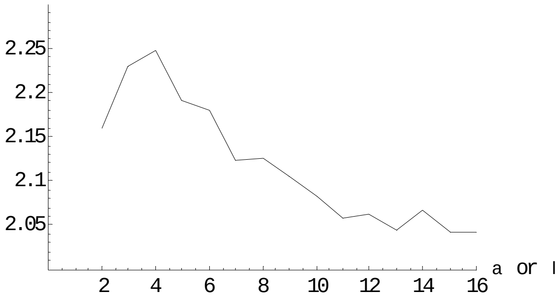
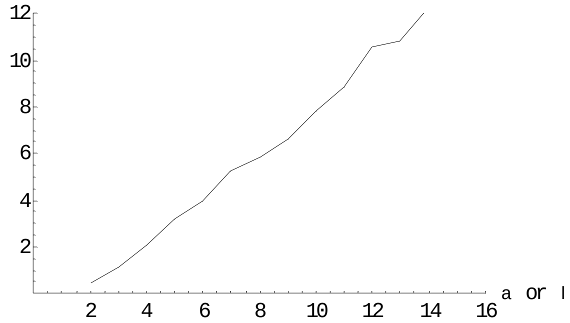


Figure 9-b : $E_\wp(\tau)$ as a function of α or λ

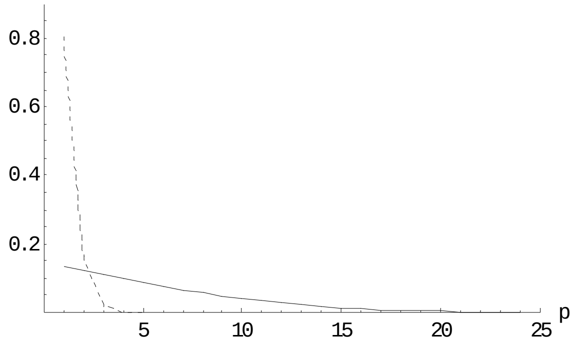


4 - Comparison between a class of single automata and a class of a chain of automata.

We give the expected limit cycle length $E_{\wp}$ (p) (Fig. 10-a) :

- Continuous line for a class of single self-modifying automata with m $\leq$ 81. 10,000 calculations.
- Doted line for a class of self-modifying automata chains. Each chain includes 3 self-modifying automata (λ=3), each automaton has 3 possible functions, card $[F_A]$=3, α^{λ} = 27. The input has 3 different possible values, card [A]= 3 and m $\leq \alpha^{\lambda}$, m$\leq$ 81. 10,000 simulations.

Figure 10-a : $E_{\wp}(p)$ as a function of p.

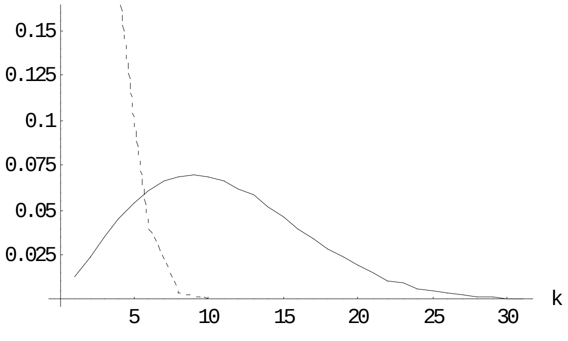


The performances of the chain of automata are higher than a single automaton : $E_{\wp}(1)$ is 80.86 % for the chain and $E_{\wp}(1)$ is 13.52 % for the single automaton.

We give the expected stabilization time $E_{\wp}$ (k) (Fig. 10-b) :

k = τ + p, τ and p are respectively the transient length and the limit cycle length.

Figure 10-b : $E_{\wp}(k)$ as a function of k.



In the table, the performances of the self-modifying automata are compared with those of the chains of automata, which have the same internal number of states.

	Self modifying automata	Chains of automata
$R_{\wp}(p)$	13.52%	80.6%
$E_{\wp}(k)$	4.99	3.38
$E_{\wp}(p)$	3.94	1.22

The chains of self modifying automata are clearly more efficient than single self-modifying automata which have the same number of internal states: a larger number of fixed points and a shorter transient length. Most of the single automata are stabilised after 29 steps and the chains after 9 steps.

Contrary to the single self-modifying automata, the chains of automata have higher performances when the number of internal states increases: The expected limit-cycle length diminishes when the number of states increases. The expected transient length increases in a linear way when α^{λ} increases, by definition, in an exponential way. The maximum value of the variable on the figures 8a and 8b are $\alpha = \lambda$ = 15, which corresponds to α^{λ} = 15^{15} = $4{,}37.10^{17}$ points of trajectory.

5 - Conclusions

Self-modifying automata and chains of self-modifying automata are non-finalist modifiable processes whenever their output value changes.

These machines after any transient steps stabilise when they go indefinitely trough a loop called p-cycle or limit cycle of length p.

Chains of automata have better performances than single self-modifying automata : Higher frequency of fixed point and a shorter transient length. The performances of the chains of automata improve when the value of their internal states increases: it is the contrary for single automata.

More often than not, the p-cycle is equal to one and is called fixed point.

In this case the external value (v) can be considered as the index of function f such as : $f_v(v) = v$ and the machine has the property of **self replication** and to be **self referential**.

In the next paper, we will describe machines that stabilise only in fixed point. We call such machine **self programming automata**.

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